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TORQUE REQUIREMENTS OF AN IN-FLIGHT SPIN MECHANISM  
FOR A PRESCRIBED MISSILE ROTATIONAL VELOCITY

12 November 1962



U S ARMY MISSILE COMMAND  
REDSTONE ARSENAL, ALABAMA

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TORQUE REQUIREMENTS OF AN IN-FLIGHT SPIN  
MECHANISM FOR A PRESCRIBED MISSILE ROTATIONAL VELOCITY

by

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Department of the Army Project No. 1-B-2-22901-D-209  
Army Material Command Management Structure Code No. 5221.11.178

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### ABSTRACT

A method has been developed to calculate the torque required to spin a small missile to a specified rotational velocity. The origin of the torque forces is assumed to be a mechanical fixture which utilizes the missile motor exhaust and, therefore, operates as an in-flight spin mechanism. The essential missile parameters are discussed and incorporated in a mathematical treatment which results in algebraic formulae for the calculation of torque.

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# LIST OF SYMBOLS

$F$	-	Net thrust causing translation.....	lbs
$T$	-	Net torque causing rotation.....	lb-ft
$I$	-	Mass moment of inertia about the roll axis of the missile.....	slug-ft <sup>2</sup>
$m$	-	Mass of missile.....	slugs
$m_0$	-	Initial mass of missile.....	slugs
$\dot{m}$	-	Time rate of change of the mass of the missile due to propellant burning.....	slugs/sec
$v$	-	Velocity of missile.....	ft/sec
$\omega$	-	Angular velocity.....	radian/sec
$s$	-	Distance along path transversed by missile...	ft
$d$	-	Distance missile travels while torque forces act.....	ft
$t$	-	Time.....	sec
$t_s$	-	Time for the missile to travel the distance s from its static position on the launcher...	sec
$t_d$	-	Time for the missile to travel the distance d from its static position on the launcher...	sec
$t_1$	-	Thrust, torque, and mass flow rate build-up time.....	sec
$a$	-	Acceleration of missile.....	ft/sec/sec

Symbols having subscript c are used to denote constant values of parameters. Symbols followed by (t) are used to denote time dependence of the variables.

## INTRODUCTION

Small missiles are often stabilized by spinning them about their longitudinal axis. For optimum design of such missile structures, the torque required to produce a given spin rate must be known. The purpose of this study is to develop a method for calculating the maximum torque necessary from an in-flight spin mechanism to produce a specified missile rotational velocity.

In this analysis, it is assumed that a known rotational velocity is to be attained in the time it takes a missile to travel a given distance along its path. Torque forces act continuously during this time and are provided by the exhaust of the motor used for translation. Thus, a direct relationship between thrust, torque, and translation is established. The torque is calculated in terms of the known missile parameters: initial mass, thrust, propellant mass flow rate, roll moment of inertia, and their time variation.

## DISCUSSION

In order to simplify the analysis and yet maintain enough generality so that it is applicable to a large number of cases, certain assumptions and considerations are made.

First, it is assumed that the required spin rate is attained very soon after ignition of the missile motor. That is, the missile simultaneously reaches a prescribed rotational velocity and translational distance within a period of time that is of the same order of magnitude as the thrust build-up time. Thus, thrust build-up time is a significant part of the total flight time in which we are interested and must be considered. A linear variation is chosen for thrust during this build-up period because it simplifies calculation and, in fact, approximates many actual motor thrust-time traces. After build-up occurs, the thrust is assumed to remain constant for the duration of flight under consideration.

Secondly, because propellant mass flow rate and thrust are directly related, they should follow the same form of variation. Therefore, a linear variation is also used for mass flow rate. It is further assumed that thrust and mass flow rate have the same build-up period so that they reach maximum values at the same time.

Thirdly, because the spin system is to utilize the motor exhaust to provide the torque forces, it is assumed that torque has the same variation as thrust and mass flow rate. Any time lag between maximum thrust and maximum torque is assumed negligible. Therefore, torque has exactly the same build-up time as thrust and mass flow rate.

The assumptions for thrust, propellant mass flow rate and torque are graphically illustrated in Figure 1. Note that all three parameters reach maximum values at time  $t_1$  and remain constant thereafter.

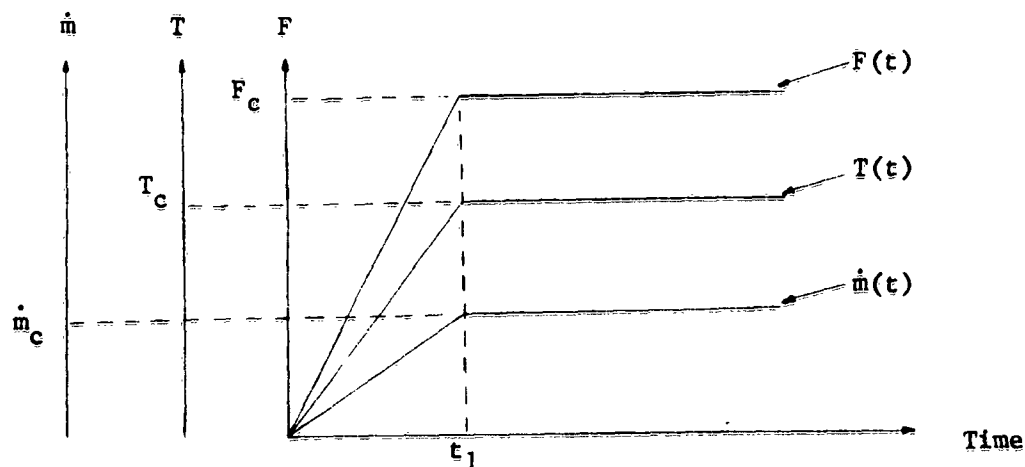


Figure 1. Net Thrust, Net Torque, & Propellant Mass Flow Rate Versus Time

In addition to the above considerations, all resistive forces, such as aerodynamic drag and friction between launcher and missile, are assumed negligible and do not enter into the analysis. Even though resistive forces might be significant in some cases, no great restriction is placed on the general method. These forces could be taken into account, if necessary, by entering the proper terms into the force and torque equations of the development in Appendices A and B.

The mathematical treatment results in two equations for the maximum torque required to produce a given rotational velocity,  $\omega$ , during the time the missile travels a given distance,  $d$ . The equation to be used depends on whether the time,  $t_d$ , required for the missile to reach the predetermined distance,  $d$ , is less than or greater than the torque build-up time,  $t_1$ . If  $t_d$  is equal to  $t_1$ , then either equation may be used. These equations are (38) and (42) in Appendix B:

$$T_{\max} = \frac{2I(t_d) \omega(t_d)}{t_d}, \quad \text{if } 0 \leq t_d \leq t_1; \quad (1)$$

and

$$T_{\max} = \frac{2I(t_d) \omega(t_d)}{2t_d - t_1}, \quad \text{if } t_d \geq t_1. \quad (2)$$

Values of torque other than maximum may be found by applying equation (37) or (42) in Appendix B for the proper time interval.

The quantity  $t_d$ , in equations (1) and (2) above, is arrived at through the use of the following two equations [equations (14) and (30) in Appendix A]:

$$s(t) = -\frac{F_c t}{\dot{m}_c} + F_c \sqrt{\frac{m_0 t_1}{2(\dot{m}_c)^3}} \ln \frac{\sqrt{2 m_0 t_1} + t \sqrt{\dot{m}_c}}{\sqrt{2 m_0 t_1} - t \sqrt{\dot{m}_c}}, \quad 0 \leq t \leq t_1 \quad (3)$$

$$s(t) = -\frac{F_c t}{\dot{m}_c} + F_c \sqrt{\frac{m_0 t_1}{2(\dot{m}_c)^3}} \ln \frac{\sqrt{2 m_0} + \sqrt{\dot{m}_c t_1}}{\sqrt{2 m_0} - \sqrt{\dot{m}_c t_1}} +$$

$$\frac{F_c m_0}{(\dot{m}_c)^2} \ln \left( \frac{2 m_0 - \dot{m}_c t_1}{2 m_0 + \dot{m}_c t_1 - 2 \dot{m}_c t} \right), \quad t \geq t_1. \quad (4)$$

Since neither equation (3) nor (4) can be easily solved for  $t$ ,  $t_d$  cannot be found explicitly. This was overcome by programming the equations on a digital computer. (Program in Appendix C). For a given missile, the set of parameters  $m_0$ ,  $\dot{m}_0$ ,  $t_1$ ,  $d$ , and  $F_c$  are known. Using these values in the program and by incrementing time from zero to a time greater than  $t_d$ , the computer will calculate and print values of distance and the elapsed time. Then if distance is plotted as a function of time,  $t_d$  may be read from the graph with considerable accuracy and can be used in equation (1) or (2).

The roll moment of inertia,  $I$ , of a missile is time dependent and factors such as missile configuration, propellant grain, and mass flow rate should be taken into consideration when calculating  $I$  at a given time.  $I(t_d)$ , in equations (1) and (2), is the moment of inertia at time  $t_d$  and must be found by a separate calculation. Formulae for mass moment of inertia may be found in handbooks and the method of decomposition can be used to calculate the moment of inertia of the given missile.

$\omega(t_d)$ , in the torque equations, is the predetermined angular velocity to be attained by the missile in time  $t_d$ . This value of  $\omega$  is assumed known for a given case. Therefore, all values in equations (1) and (2) are known. The maximum torque can then be found by performing the indicated mathematical operations.

Appendix D presents an application of the method to an example problem.

## APPENDIX A

### EQUATIONS OF TRANSLATIONAL MOTION

The following curves represent graphically the assumed time variation of thrust, propellant mass flow rate, rate of change of the mass of the missile, and the mass of the missile:

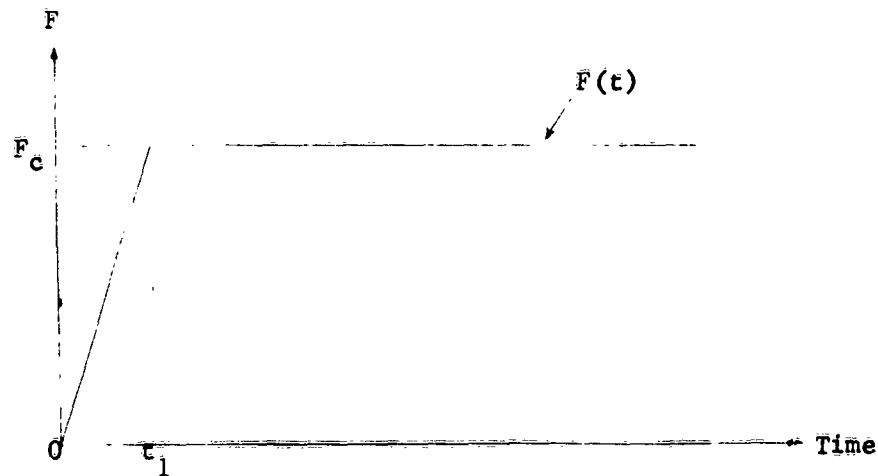


Figure 2. Thrust Versus Time

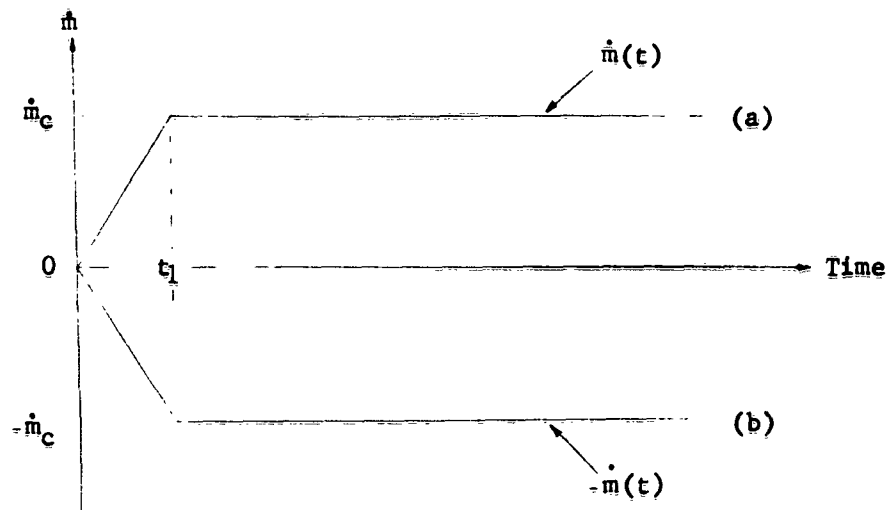


Figure 3. (a) Propellant Mass Flow Rate Versus Time  
(b) Rate of Change of Missile Mass Versus Time

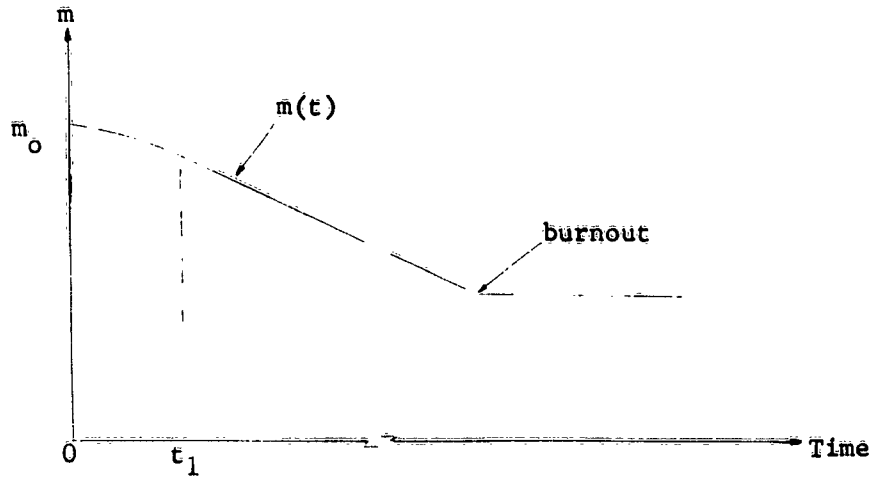


Figure 4. Missile Mass Versus Time

From Newton's second law of motion, we have

$$F = \frac{d(mv)}{dt}. \quad (5)$$

In the present application, both  $F$  and  $m$  are time dependent as shown in Figures 2 and 4, respectively. For the time interval  $0 \leq t \leq t_1$  in Figure 2,

$$F(t) = \frac{t}{t_1} F_c. \quad (6)$$

Using equation (6) in equation (5), it follows that

$$\int_0^{m(t)v} d(mv) = \int_0^t F(t) dt = \frac{F_c}{t_1} \int_0^t t dt; \quad (7)$$

or after integrating,

$$m(t)v = \frac{t^2}{2t_1} F_c. \quad (8)$$

To evaluate the quantity  $m(t)$ , consider curve b of Figure 3 over the time interval  $0 \leq t \leq t_1$ . The rate of change of the mass of the missile is given by

$$\dot{m} = \frac{dm}{dt} = - \frac{t}{t_1} \dot{m}_c. \quad (9)$$

Then, by separating variables and integrating we obtain

$$\int_{m_0}^{m(t)} dm = - \frac{\dot{m}_c}{t_1} \int_0^t t dt; \quad (10)$$

or

$$m(t) = m_0 - \dot{m}_c \frac{t^2}{2t_1} \quad (11)$$

This is the expression for the mass of the missile at any time  $t$  in this time interval. Combining equation (11) with equation (8) gives

$$v = \frac{ds}{dt} = \frac{F_c t^2}{2t_1 m_0 - \dot{m}_c t^2}, \quad (12)$$

which is the equation of velocity in the time interval  $0 \leq t \leq t_1$ .

The equation for distance as a function of time is found by separating variables in equation (12) and integrating as follows:

$$\int_0^s ds = F_c \int_0^t \frac{t^2}{2t_1 m_0 - \dot{m}_c t^2} dt$$

or

$$s(t) = F_c \left[ - \frac{t}{\dot{m}_c} + \frac{m_0 t_1}{\sqrt{2m_0 t_1} (\dot{m}_c)^{3/2}} \ln \frac{\sqrt{2m_0 t_1} + t \sqrt{\dot{m}_c}}{\sqrt{2m_0 t_1} - t \sqrt{\dot{m}_c}} \right]. \quad (13)$$

After rearranging, equation (13) becomes

$$s(t) = - \frac{F_c t}{\dot{m}_c} + F_c \sqrt{\frac{m_0 t_1}{2(\dot{m}_c)^3}} \ln \frac{\sqrt{2m_0 t_1} + t \sqrt{\dot{m}_c}}{\sqrt{2m_0 t_1} - t \sqrt{\dot{m}_c}}. \quad (14)$$

Equation (14) is the relationship between distance and time in the interval  $0 \leq t \leq t_1$ . When  $t = t_1$ , equation (14) becomes

$$s(t) = s(t_1) = - \frac{F_c t_1}{\dot{m}_c} + F_c \sqrt{\frac{m_0 t_1}{2(\dot{m}_c)^3}} \ln \frac{\sqrt{2m_0 t_1} + t_1 \sqrt{\dot{m}_c}}{\sqrt{2m_0 t_1} - t_1 \sqrt{\dot{m}_c}}. \quad (15)$$

For the time interval  $t \geq t_1$  in Figure 2,

$$F(t) = F_c. \quad (16)$$

Using equation (16) in equation (5), it follows that

$$\frac{m(t)v}{m(t_1)v(t_1)} = \int_{t_1}^t F(t)dt = F_c \int_{t_1}^t dt; \quad (17)$$

or

$$m(t)v = F_c(t - t_1) + m(t_1)v(t_1). \quad (18)$$

From curve b of Figure 3 over the time interval  $t \geq t_1$ ,

$$\dot{m} = \frac{dm}{dt} = -\dot{m}_c. \quad (19)$$

Thus,

$$\frac{m(t)}{m(t_1)} = -\dot{m}_c \int_{t_1}^t dt; \quad (20)$$

or

$$m(t) = m(t_1) - \dot{m}_c (t - t_1). \quad (21)$$

The value  $m(t_1)$ , in equation (21), can be found from equation (11) by setting  $t = t_1$ . This gives

$$m(t_1) = m_0 - \dot{m}_c \frac{t_1}{2}. \quad (22)$$

Substituting this quantity into equation (21) and simplifying, we obtain

$$m(t) = m_0 + \dot{m}_c \frac{t_1}{2} - \dot{m}_c t, \quad (23)$$

which is the equation for the mass of the missile at any time  $t$  in the interval  $t \geq t_1$ .

By combining equation (23) with equation (18) and rearranging we arrive at

$$v = \frac{ds}{dt} = \frac{F_c(t - t_1) + m(t_1)v(t_1)}{m_0 + \dot{m}_c \frac{t_1}{2} - \dot{m}_c t}. \quad (24)$$

The factor  $m(t_1)v(t_1)$  can be evaluated by taking the product of equations (22) and (12) with  $t = t_1$ . This gives

$$m(t_1)v(t_1) = \frac{F_{ct_1}}{2}. \quad (25)$$

Finally, when equation (25) is combined with equation (24), we have

$$v = \frac{ds}{dt} = \frac{F_{ct} - \frac{F_{ct_1}}{2}}{m_0 + \frac{\dot{m}_c t_1}{2} - \dot{m}_c t}, \quad (26)$$

which is the equation of velocity in the time interval  $t \geq t_1$ .

Now, for convenience of handling equation (26), set the constant

$$p = m_0 + \frac{\dot{m}_c t_1}{2}. \quad (27)$$

Then, after separating variables and substituting equation (27) into equation (26), we obtain

$$ds = F_c \frac{tdt}{p - \dot{m}_c t} - \frac{F_{ct_1} dt}{2(p - \dot{m}_c t)}. \quad (28)$$

The equation for distance as a function of time in the interval  $t \geq t_1$  is found from equation (28) by integrating distance from  $s(t_1)$  to  $s(t)$  and time from  $t_1$  to  $t$ . This gives

$$\begin{aligned} s(t) - s(t_1) = F_c \left[ -\frac{t}{\dot{m}_c} - \frac{p}{(\dot{m}_c)^2} \ln(p - \dot{m}_c t) + \frac{t_1}{\dot{m}_c} \right. \\ \left. + \frac{p}{(\dot{m}_c)^2} \ln(p - \dot{m}_c t_1) \right] - \frac{F_{ct_1}}{2\dot{m}_c} \left[ -\ln(p - \dot{m}_c t) \right. \\ \left. + \ln(p - \dot{m}_c t_1) \right]. \end{aligned} \quad (29)$$

Now, by adding equation (15) to both sides of equation (29), substituting (27) into (29), and simplifying, we have

$$\begin{aligned} s(t) = -\frac{F_{ct}}{\dot{m}_c} + \frac{F_c m_0}{(\dot{m}_c)^2} \ln \left( \frac{2m_0 - \dot{m}_c t_1}{2m_0 + \dot{m}_c t_1 - 2\dot{m}_c t} \right) \\ + F_c \sqrt{\frac{m_0 t_1}{2(\dot{m}_c)^3}} \ln \frac{\sqrt{2m_0 + \dot{m}_c t_1}}{\sqrt{2m_0 - \dot{m}_c t_1}}. \end{aligned} \quad (30)$$

This equation gives the distance traveled by the missile beyond  $s(t_1)$  in time  $t$ , where  $t \geq t_1$ .

The acceleration term for the time interval  $0 \leq t \leq t_1$ , is obtained by differentiating equation (12) with respect to time. This gives

$$a = \frac{d^2s}{dt^2} = \frac{4F_c \dot{m}_0 t_1 t}{(2m_0 t_1 - \dot{m}_c t^2)^2}. \quad (31)$$

Similarly, differentiation of equation (26) gives the acceleration term for the time interval  $t \geq t_1$ . The result is

$$a = \frac{d^2s}{dt^2} = \frac{4F_c \dot{m}_0}{(2m_0 + \dot{m}_c t_1 - 2\dot{m}_c t)^2}. \quad (32)$$

## APPENDIX B

### EQUATIONS OF ROTATIONAL MOTION

Equations for calculating the torque necessary to produce a given rotational velocity during the time a missile has traversed a given distance,  $d$ , are developed in this appendix. The assumed time history of the applied torque is given in Figure 5.

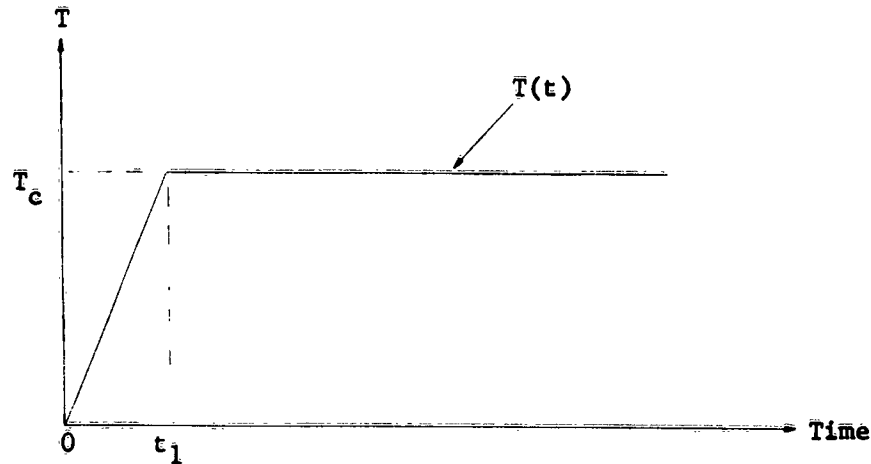


Figure 5. Torque Versus Time

From the rotational analog of Newton's second law, we have

$$T = \frac{d(I\omega)}{dt},$$

or

$$Tdt = d(I\omega). \quad (33)$$

For the case shown in Figure 5, torque, in the time interval  $0 \leq t \leq t_1$ , is given by

$$T(t) = \frac{T_c}{t_1}t. \quad (34)$$

By substituting equation (34) into equation (33), we obtain

$$\frac{T_c}{t_1} \int_0^t t dt = \int_0^{I(t)\omega(t)} d(I\omega) \quad (35)$$

or, after performing the integration and rearranging, equation (35) gives

$$T_c = \frac{2t_1 I(t) \omega(t)}{t^2}. \quad (36)$$

Then, substituting equation (36) into equation (34),

$$T(t) = \frac{2I(t) \omega(t)}{t}, \quad (37)$$

which is the expression for torque at any time  $t$  when  $0 \leq t \leq t_1$ .

Now, it is seen that if  $0 \leq t = t_d \leq t_1$  in Figure 5, then  $T(t) = T(t_d) = T_{\max}$ . Thus, from equation (37), the maximum torque that must be reached in time  $t_d$  to provide a rotational velocity,  $\omega(t_d)$ , is given by

$$T_{\max} = \frac{2I(t_d) \omega(t_d)}{t_d}, \quad (38)$$

if  $0 \leq t_d \leq t_1$  and torque varies as in Figure 5.

For the time interval  $t \leq t_1$ ,

$$T(t) = T_c. \quad (39)$$

Applying equation (33) to this case and including the effect of torque throughout the interval  $0 \leq t \leq t_1$ , we obtain

$$\frac{T_c}{t_1} \int_0^{t_1} t dt + T_c \int_{t_1}^t dt = \int_0^t d(I\omega) \quad (40)$$

After performing the integration and rearranging, we have

$$T_c = \frac{2I(t) \omega(t)}{2t - t_1}. \quad (41)$$

Now, it is seen that if  $t = t_d \geq t_1$  in Figure 5, then

$$T(t) = T_c = T_{\max}.$$

Thus, from equation (41), the maximum torque that must be reached in time  $t_d$  to provide a rotational velocity  $\omega(t_d)$  is given by

$$T(t) = T_{\max} = \frac{2I(t_d) \omega(t_d)}{2t_d - t_1} \quad (42)$$

if  $t_d \geq t_1$  and torque varies as in Figure 5.

## APPENDIX C

### IBM 1620 FORTRAN COMPUTER PROGRAM

The purpose of this program is to obtain data for plotting translational distance, velocity, and acceleration as functions of time. The plot of distance versus time may be used for finding  $t_d$  as outlined in the DISCUSSION. The velocity and acceleration plots are not used in the method for calculating torque and are included only for completeness.

Equations expressing the relationship of distance, velocity, and acceleration with time are taken from Appendix A. They are as follows:

For the time interval  $0 \leq t \leq t_1$ .

$$s(t) = -\frac{F_c t}{\dot{m}_c} + F_c \sqrt{\frac{m_0 t_1}{2(\dot{m}_c)^3}} \ln \frac{\sqrt{2m_0 t_1} + t\sqrt{\dot{m}_c}}{\sqrt{2m_0 t_1} - t\sqrt{\dot{m}_c}} \quad (14)$$

$$v = \frac{F_c t^2}{2m_0 t_1 - \dot{m}_c t^2} \quad (12)$$

$$a = \frac{4F_c m_0 t_1 t}{(2m_0 t_1 - \dot{m}_c t^2)^2} \quad (31)$$

And for the time interval  $t \geq t_1$ .

$$s(t) = -\frac{F_c t}{\dot{m}_c} + \frac{F_c m_0}{(\dot{m}_c)^2} \ln \left( \frac{2m_0 - \dot{m}_c t_1}{2m_0 + \dot{m}_c t_1 - 2\dot{m}_c t} \right) + F_c \sqrt{\frac{m_0 t_1}{2(\dot{m}_c)^3}} \ln \frac{\sqrt{2m_0} + \sqrt{\dot{m}_c t_1}}{\sqrt{2m_0} - \sqrt{\dot{m}_c t_1}} \quad (30)$$

$$v = \frac{F_c t - \frac{F_c t_1}{2}}{m_0 + \frac{\dot{m}_c t_1}{2} - \dot{m}_c t} \quad (26)$$

$$a = \frac{4F_c m_0}{(2m_0 + \dot{m}_c t_1 - 2\dot{m}_c t)^2} \quad (32)$$

The computer program is written in Fortran language with format control and intended for use on an IBM 1620 digital computer with type-writer output capability. Since it is desirable to have the print out data in conventional form, all quantities are programmed in floating point mode. The following floating point symbols are used:

$A = \dot{m}_0$	$DD =$	increment of D
$B = \dot{m}_c$	$DX =$	increment of X
$T = F_c$	$DT =$	increment of T
$X = t$	$DMAX =$	maximum D
$D = t_1$	$TMAX =$	maximum T
$SD = d$		

To allow investigation of several cases for one missile configuration, the program requires that input data include values for incrementing net thrust,  $DT$ , and net thrust build-up time,  $DD$ . These values are set equal to zero when only one case is considered. Also, input data must include the maximum thrust build-up time,  $DMAX$ , and the maximum thrust,  $TMAX$ , that is to be considered for a single configuration. For proper program operation,  $DX$ , the time increment at which calculations are to be made, must be of such value that a multiple of  $DX$  equals  $D$ . Other input data may be determined by comparing the READ statements of the program to the above list of floating point symbols. Sense Switch "One" is programmed "on" to permit a complete change of input data corresponding to a change in missile configuration.

Control statements are written into the program so that the computer automatically selects the correct equation for calculating distance, velocity, and acceleration. This is done by checking the total elapsed time against the build-up time after each time increment,  $DX$ , has been added. If  $t \leq t_1$  and  $s \leq d$ , the program will make calculations using equations (14), (12), and (31) of Appendix A. If  $s > d$  at some point in the time interval  $0 \leq t \leq t_1$ , the computer will stop. If, however,  $s < d$  when  $t = t_1$ , the computer will begin calculating with equations (30), (26), and (32) of Appendix A, and continue operation until the condition,  $s = d + 2$  feet, is satisfied.

Data output of thrust, thrust build-up time, elapsed time, distance, velocity, and acceleration is printed in columnar form. Column headings are automatically printed out by the computer typewriter. A copy of the complete program follows:

DT MUST BE OF SUCH VALUE THAT A MULTIPLE OF DT EQUALS

```

D .
149 FORMAT(49H      TIME      DISTANCE  VELOCITY  ACCELER
ATION )
150 FORMAT(F7.4,F7.4,F9.2,F7.2,F6.3,F9.3)
151 FORMAT(F6.3,F5.3,F5.3,F5.3,F9.2)
152 FORMAT(F9.2)
153 FORMAT(F5.3)
154 FORMAT(F11.4,F11.4,F13.4,F14.4)
156 FORMAT(F9.2,F12.2)
25 READ 150,X,DX,T,DT,A,SD
    READ 151,B,D,DD,DMAX,TMAX
4 PRINT 152,T
3 PRINT 153,D
  PRINT 149
2 VC=T*X*X
  VR= 2.0*D*A - B*X*X
  VC= VC/VR
  AC= 4.0*D*A*T*X
  AC=AC/(VR*VR)
  E=(2.*A)*D
  E=E**.5
  F=X*(B**.5)
  G=(E+F)/(E-F)
  IF(G)9,9,22
9 PRINT 156,T,T
  GO TO 12
22 G=LCG(G)
  H=(A*D)/(2.*(D**3.))
  H=H**.5
  SA=-T*X/B+T*(H*G)
  IF(X-D)8,7,7
7 PRINT 154,X,SA,VC,AC
  IF(SA-SD)90,90,60
8 PRINT 154,X,SA,VC,AC
  SS=SD+2.0
  IF(SA-SS)12,60,60
12 X=X+DX
  GO TO 2
90 VZ= (2.0*T*X) - (T*D)
  VY= 2.0*A + B*D - 2.0*B*X
  VZ= VZ/VY
  EC=4.0*A*T
  EB=VY*VY
  EC= EC/EB
  S=T*(D-X)/B
  P=(2.*A-B*D)/(2.*A+B*D-2.*B*X)
  P=LCG(P)
  S=S+(T*A/B**2.)*P+SA
  IF(SENSE SWITCH 1)25,21

```

```

21 PRINT 154,X,S,VZ,EC
   IF(S=SS)50,60,60
50 X=X+DX
   GC TC 90
60 IF(D=DMAX)40,30,30
40 D=D+DD
   X=0.0
   GC TC 3
30 IF (T=TMAX)200,100,100
200 T=T+DT
   D=D-DMAX+DD
   X=0.0
   GC TC 4
100 PAUSE
   GC TC 25
   END

```

## APPENDIX D

### AN ILLUSTRATIVE EXAMPLE

To illustrate the method for computing the maximum value of torque required of an in-flight spin mechanism in producing a certain distance dependent missile spin rate, the following example is presented:

#### PROBLEM:

A missile, with an initial mass of 0.327 slugs, must be accelerated to an angular velocity of 800 radians per second in the time required for the missile to travel  $3 \frac{2}{3}$  feet from the static position on its launcher. The motor supplies a constant net thrust of 4208 pounds after a linear build-up in 0.02 seconds. The propellant mass flow rate reaches a value of 0.545 slugs per second in 0.02 seconds and remains constant thereafter. Determine the maximum torque necessary to produce the required angular velocity.

#### SOLUTION:

The given data is:

$$m_0 = 0.327 \text{ slugs}$$

$$F_c = 4208 \text{ pounds}$$

$$\dot{m}_c = 0.545 \text{ slugs/sec}$$

$$t_1 = 0.02 \text{ seconds}$$

$$d = 3.667 \text{ feet}$$

$$\omega(t_d) = 800 \text{ radians/sec}$$

The computer program in Appendix C may be employed to obtain the data listed in Table I. A plot of the distance versus time values yields the curve of Figure 6.

TABLE I

## 1620 COMPUTER DATA PRINTOUT FOR EXAMPLE PROBLEM

 $F_c = 4208$  pound $t_1 = .020$  seconds

<u>Time (sec)</u>	<u>Distance (ft)</u>	<u>Velocity (ft/sec)</u>	<u>Acceleration (ft/sec<sup>2</sup>)</u>
.0000	.0000	.0000	.0000
.0025	.0015	2.0112	1609.4007
.0050	.0133	8.0512	3223.8381
.0075	.0452	18.1388	4848.3880
.0100	.1074	32.3058	6488.2065
.0125	.2101	50.5969	8148.5689
.0150	.3639	73.0703	9834.9162
.0175	.5791	99.7979	11552.8960
.0200	.8665	130.8661	13308.4190
.0200	.8665	130.8661	13308.4180
.0225	1.2353	164.2787	13421.9220
.0250	1.6881	197.9769	13536.8850
.0275	2.2253	231.9644	13653.3300
.0300	2.8478	266.2448	13771.2850
.0325	3.5568	300.8221	13890.7750
.0350	4.3523	355.7000	14011.8270
.0375	5.2353	370.8825	14134.4690
.0400	6.2071	406.3737	14258.7270

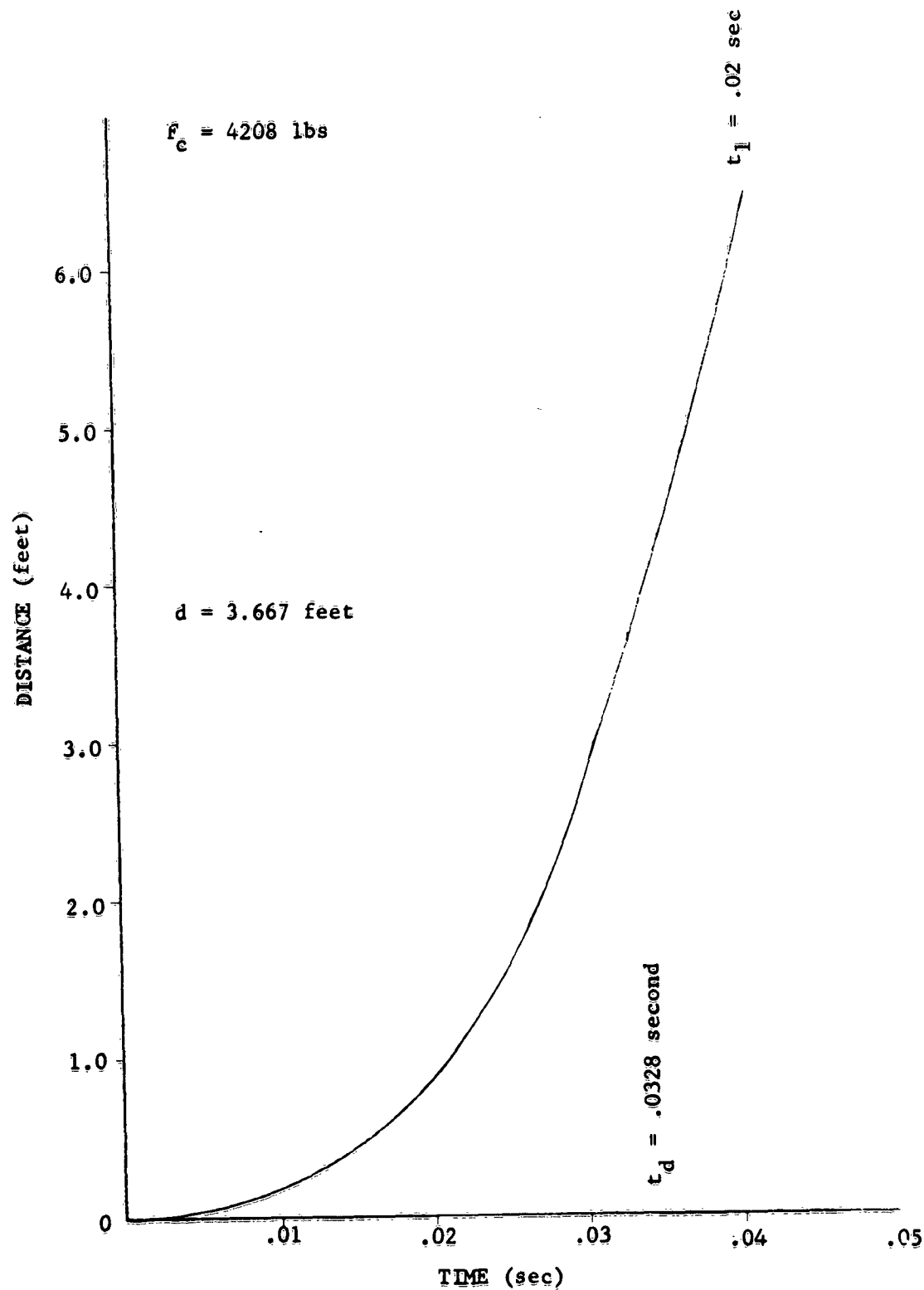


Figure 6. Translational Distance Versus Time

The time,  $t_d$ , required for the missile to travel the distance  $d$ , can be read from the abscissa of Figure 6 as .0328 seconds. Since this value of  $t_d$  is greater than  $t_1$  (.02 seconds), equation (41) in Appendix B is used to compute the maximum torque:

$$T_{\max} = \frac{2 I(t_d) \omega(t_d)}{2 t_d - t_1}$$

In practice, the moment of inertia,  $I(t_d)$ , must be calculated before equation (41) can be applied. This will require a knowledge of the missile configuration, propellant grain, mass flow rate, and other pertinent factors. To complete the example, assume that the calculation gives  $I(t_d) = 2.458 \times 10^{-3}$  slug-ft<sup>2</sup>. Then, applying equation (41),

$$T_{\max} = \frac{2 (2.458 \times 10^{-3} \text{ slug ft}^2) (800 \text{ rad/sec})}{2 (.0328 \text{ sec}) - .02 \text{ sec}}$$

$$T_{\max} = 86.245 \text{ ft-lbs.} = 1035 \text{ inch-lbs.}$$

which is the maximum torque necessary to produce a spin rate of 800 rad/sec for this hypothetical missile.

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